Journal of Applied Mechanics and Technical Physics, Vol. 51, No. 2, pp. 218–225, 2010

NUMERICAL DESIGN OF MULTIMODAL AXISYMMETRIC HYPERSONIC NOZZLES FOR WIND TUNNELS

S. M. Aulchenko,[1] V. M. Galkin,[2]
V. I. Zvegintsev,[1] and A. N. Shiplyuk[1]

UDC 533.697

A method for design of hypersonic nozzles for wind tunnels is developed and implemented on the basis of solving direct problems with various models of the medium and numerical methods of integration of gas-flow equations. Multimodal nozzles for operation in Mach number ranges $M_{\rm out} = 8$–14 *and* $M_{\rm out} = 14$–20 *satisfying specified requirements are designed.*

Key words: *hypersonic nozzle, Navier–Stokes equations, Euler equations, method of characteristics, numerical optimization.*

Introduction. It is known that a good quality of the wind-tunnel flow is ensured by using carefully designed nozzles accelerating the gas flow to a specified velocity. Almost all existing wind tunnels use unimodal nozzles designed for certain operation conditions. As such nozzles are manufactured with specialized equipment providing precision accuracy and are extremely expensive, they are limited in number; therefore, the number of wind-tunnel operation regimes is also limited.

The AT-303 hypersonic wind tunnel based at the Khristianovich Institute of Theoretical and Applied Mechanics of the Siberian Division of the Russian Academy of Sciences is designed for operation in wide ranges of the Mach and Reynolds numbers. Therefore, the issue of using multimodal nozzles that can form flows with different parameters with the minimum changes in the nozzle structure has been discussed all the time after wind-tunnel commissioning. The present paper describes the sequence of actions and methods used in numerical design of multimodal nozzles for the AT-303 hypersonic wind tunnel.

Multimodal Nozzle Variants. A preliminary analysis of possible variants of the multimodal nozzle was based on the model of an ideal perfect gas. The most effective option in this model for a uniform flow with the output Mach number $\mathrm{M}_{\rm out}$ is the use of a three-parameter family $W(\gamma, \mathrm{M}_{\rm out}, G)$ of supersonic contours with a plane sonic line at the input and a uniform characteristic at the output (γ is the ratio of specific heats, $\mathrm{M}_{\rm out}$ is the output Mach number, and G is the ratio of the flow rate through the streamline used to construct the nozzle to the flow rate of the nozzle with a corner point). If the ambient pressure does not exceed the pressure at the nozzle exit (output pressure), the operation domain of such nozzles is shaped as a diamond, which can be obtained by means of mirror reflection of the uniform characteristic with respect to the abscissa axis and with respect to the vertical straight line passing through the end point of the nozzle. Obviously, each nozzle of this kind corresponds to one operation mode. The procedure of constructing the family of the contours $W(\gamma, \mathrm{M}_{\rm out}, G)$ by the method of characteristics was described in [1].

Variant No. 1. Using the family of the contours $W(\gamma, \mathrm{M}_{\rm out}, G)$ for two modes M_1 and M_2, we can design a nozzle consisting of a fixed part adjacent to the throat section and a replaceable part adjacent to this fixed part on the right. In this paper, we consider only a bimodal nozzle, but further construction is also applicable to multimodal nozzles as well. The nozzle is constructed as follows:

[1]Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; aultch@itam.nsc.ru. [2]Institute of Geology, Oil, and Gas at the Tomsk Polytechnical University, Tomsk 634050. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 51, No. 2, pp. 90–97, March–April, 2010. Original article submitted December 4, 2008; revision submitted June 3, 2009.

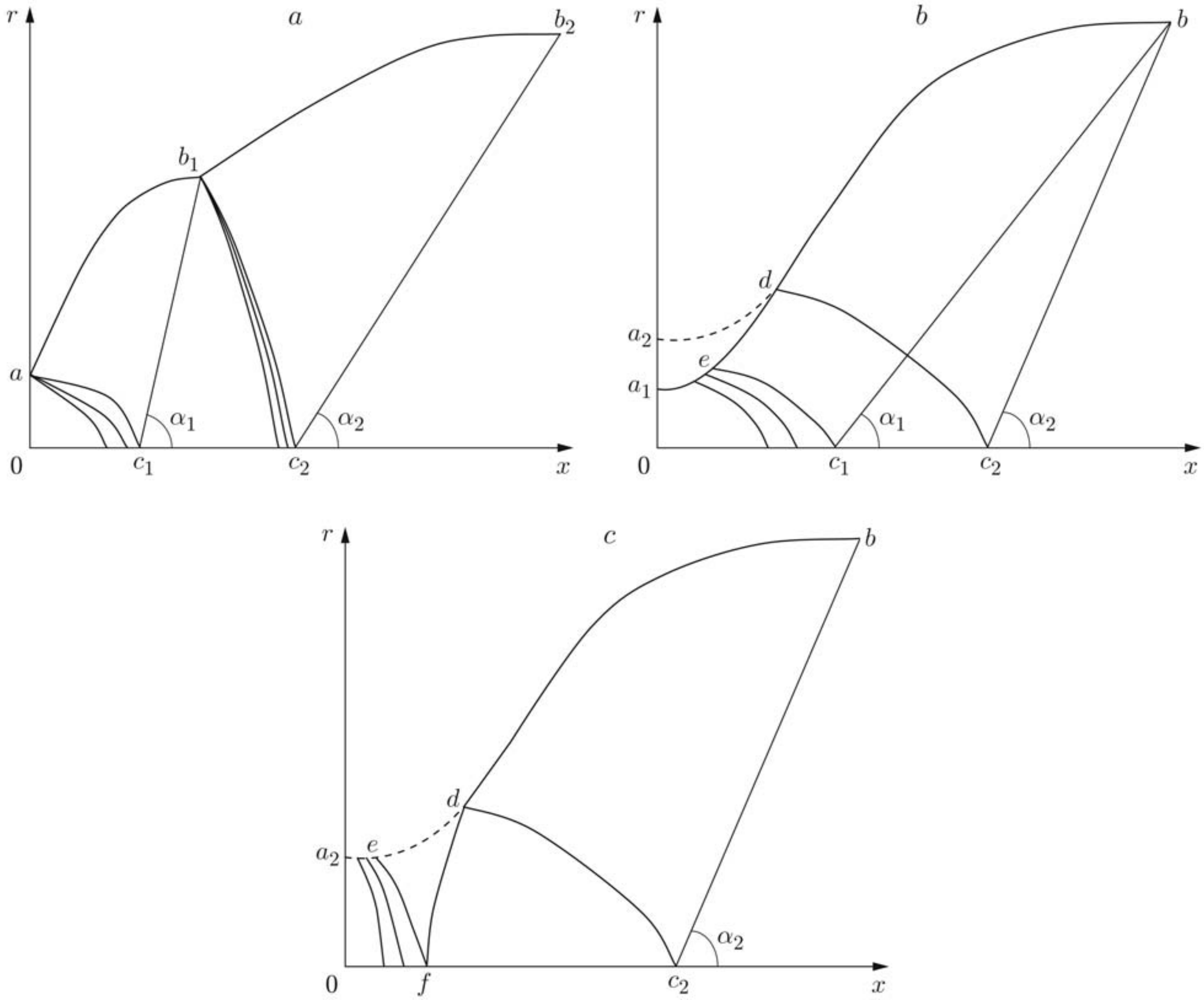

Fig. 1. Design of a multimodal nozzle: (a) variant No. 1 with corner points [ac_1 and b_1c_2 are the characteristics C^- closing the expansion fan; c_1b_1 and c_2b_2 are the uniform characteristics; $\alpha_1 = \arcsin(1/\mathrm{M}_1)$; $\alpha_2 = \arcsin(1/\mathrm{M}_2)$]; (b) variant No. 2 [$a_1d$ and a_2d are the replaceable parts for M_1 and M_2; db is the fixed part of the nozzle; dc_2 and ec_1 are the characteristics C^-; c_1b and c_2b are the uniform characteristics; $\alpha_1 = \arcsin(1/\mathrm{M}_1)$; $\alpha_2 = \arcsin(1/\mathrm{M}_2)$]; (c) variant No. 2 with the use of the solution of the Goursat problem [a_2d is the replaceable part of the nozzle for M_2; db is the fixed part of the nozzle; c_2b is the uniform characteristic; $\alpha_2 = \arcsin(1/\mathrm{M}_2)$].

1) the first (fixed) part ab_1 from the family $W(\gamma, \mathrm{M}_1, G)$ is constructed for a given Mach number M_1 ($\mathrm{M}_1 < \mathrm{M}_2$) at the nozzle exit and for a specified ratio of the gas flow rates G;

2) the second part b_1b_2 is constructed from the family $W(\gamma, \mathrm{M}_2, G)$, with the condition in the initial section being a uniform flow with the Mach number M_1 rather than a plane sonic line.

A sketch of this variant of the multimodal nozzle with corner points is shown in Fig. 1a, but the use of intermediate streamlines makes it possible to construct a smooth nozzle. By virtue of structural constraints, we further consider only smooth nozzles.

Variant No. 2. Let us consider the nozzle structure proposed in [2] for two modes M_1 and M_2 ($\mathrm{M}_1 > \mathrm{M}_2$). This nozzle consists of a fixed part and a smoothly adjacent replaceable part. The replaceable part contacts the fixed part on the left and the nozzle throat on the right. For this variant, Galkin and Zvegintsev [3] proposed to choose the contours of the fixed and replaceable parts from the family of the contours $W(\gamma, \mathrm{M}_{\mathrm{out}}, G)$, but the question about the length of the fixed part remained open. A more careful analysis [4] showed that the length of

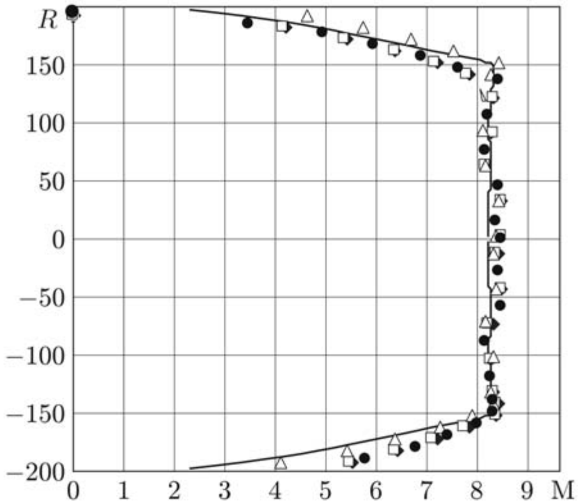

Fig. 2. Distribution of the Mach number on the nozzle exit at $M_{out} = 8$: the points and the solid curve show the experimental and computed results, respectively.

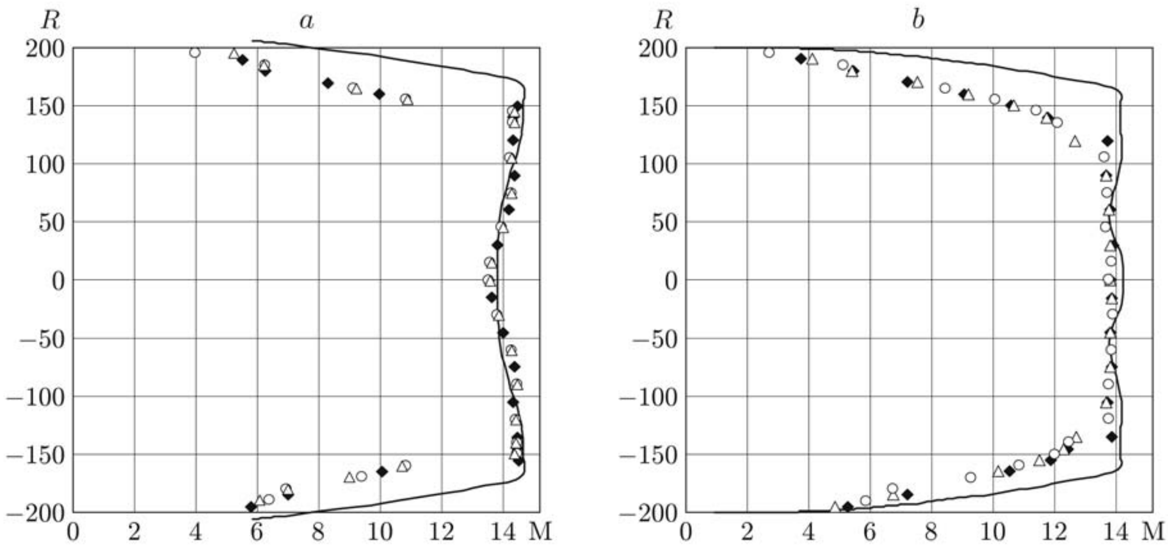

Fig. 3. Distribution of the Mach number at a distance $x = 0.4$ m from the nozzle exit (a) and at the nozzle exit (b) at $M_{out} = 14$: the points and the solid curve show the experimental and computed results, respectively.

the fixed part, as well as its shape, is uniquely calculated by the method of characteristics through the following algorithm (Fig. 1b):

1) the nozzle with $a_1b \in W(\gamma, M_1, G)$ ($\gamma = 1.4$) is constructed;

2) the uniform characteristic c_2b is constructed;

3) the parameters between the wall a_1b and the uniform characteristic c_2b are calculated by the method of characteristics until the characteristic dc_2 emanating from the point on the axis is obtained. The point d divides the nozzle a_1b into two parts: replaceable part a_1d for the Mach number M_1 and common fixed part db of the minimum length for the Mach numbers M_1 and M_2. As a further development of the method of constructing such a nozzle, it is proposed to use the method of characteristics to find the shape of the replaceable part a_2d. For this purpose, the parameters in the domain between the characteristic dc_2 constructed by the algorithm described above and the axis are calculated until the characteristic df emanating from the point d is obtained (Fig. 1c). Simultaneously, we obtain the Mach number M_f at the point f. After that, the parameters in the expansion wave

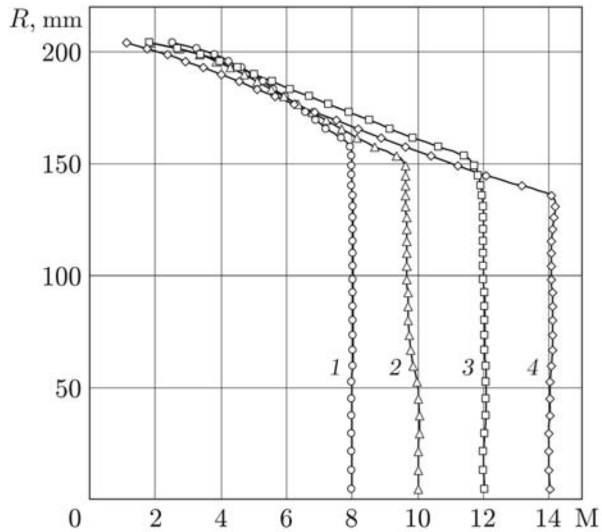

Fig. 4. Computed distributions of the Mach number at the exit of a multimodal nozzle with different output Mach numbers ($x = 3$ m): $\mathrm{M_{out}} = 8$ (1), 10 (2), 12 (3), and 14 (4).

TABLE 1

Values of ΔM and $\mathrm{M_{av}}$ in the Cross Section $x = 4.0$ m for the Nozzle with $\mathrm{M_{out}} = 8$ and Different Computational Grids

Grid	ΔM, %	$\mathrm{M_{av}}$
1000×40	0.17	8.10
2000×40	0.15	8.10
1000×80	0.17	8.11

are calculated beginning from the plane sonic line until the Mach number M_f is obtained on the axis. Thus, we find the characteristic ef closing the expansion wave. After that, the contour de is constructed between the known characteristics df and ef by choosing the parameter G and solving the Goursat problem.

A comparison of the three variants of design of the multimodal nozzle considered above shows that the advantages of variant No. 1 are the simplicity of calculations and the presence of a constant subsonic part and an adjacent supersonic part of the nozzle. The total length of the nozzle, however, is greater than that in variant No. 2. Therefore, variant No. 2 is further used as the basic variant.

In this work, as in [5], the replaceable part a_2d for the Mach number M_2 at the nozzle exit was constructed by a direct method based on solving the Euler equations and the quasi-Newton method of searching for the minimum of a function of numerous variables [6]. The use of the method of characteristics in a direct method requires significant time expenses because of the large nozzle length; therefore, the MacCormack marching scheme was used [7]. The deviation of the constructed characteristic dc_2 from the known characteristic dc_2 was used as the minimized functional. The contour was approximated by basis functions; the angle of inclination in the initial section was set equal to zero, which ensured construction of smooth contours. As the basis functions, it seems reasonable to use power functions of the form

$$f(x) = \sum_{i=1} c_i t^{i-1}, \qquad t = \frac{\ln(1 + x - x_a)}{\ln(1 + x_d - x_a)},$$

where $f(x)$ is the nozzle contour, c_i are the varied variables, x_a is the abscissa of the throat section, and x_d is the abscissa of the point of attachment to the fixed part.

TABLE 2

Range of the Reynolds Numbers at the Nozzle Exit for Different Mach Numbers

M_{out}	$Re_{min} \cdot 10^7$	$Re_{av} \cdot 10^7$	$Re_{max} \cdot 10^7$
8	0.94	3.92	11.9
10	0.49	1.32	3.02
12	0.25	0.87	2.35
14	0.19	0.90	2.94

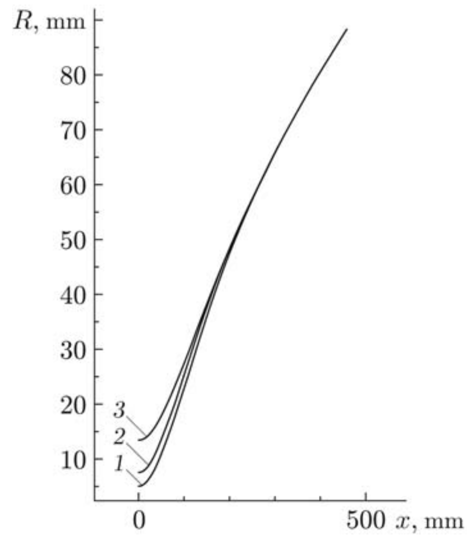

Fig. 5. Shapes of the 0.32-m replaceable part of the nozzle for different output Mach numbers: $M_{out} = 12$ (1), 10 (2), and 8 (3).

Construction of the Initial Approximation. The flow model used to design the multimodal nozzle variants discussed here ignores the viscosity and, correspondingly, the presence of the boundary layer, which exerts a significant effect on the characteristics of hypersonic nozzles [8]. Therefore, a comprehensive approach with the ideal perfect gas model and Navier–Stokes equations was used to solve the problem posed. Solving optimization problems for viscous flows requires considerable computational resources and a good initial approximation. This approximation was taken to be the nozzle contour obtained, as was mentioned above, with the model of an ideal perfect gas.

Testing of the Program. The viscous flow in the hypersonic nozzle with a prescribed shape was assumed to be computed by the Fluent CFD program; therefore, it was necessary to verify the accuracy of solving the Navier–Stokes equation and the (k–ω) model of turbulence. For this purpose, a real axisymmetric nozzle with an output diameter of 0.4 m was created with the use of the contour obtained by the method described in [9]. The output Mach numbers $M_{out} = 8$, 10, 12, and 14 were expected at the nozzle exit. The experimental and computed distributions of the Mach number at the nozzle exit and at a distance $x = 0.4$ m from the nozzle exit with the use of replaceable parts forming the nozzle throat are shown for the Mach numbers $M_{out} = 8$ and 14 in Figs. 2 and 3, respectively. The experimental technique was described in [10]. The computational grid had 1000 nodes in the axial direction and 40 nodes over the nozzle radius. To test the convergence of the solution, we performed computations with a doubled number of grid nodes. The root-mean-square deviations ΔM from the average Mach number and the average Mach numbers M_{av} in the cross section $x = 4$ m, which were computed with the use of different grids, are listed in Table 1 for the Mach number $M_{out} = 8$. It is seen that the nozzle characteristics used to calculate the minimized functional of the optimization problem remain almost unchanged as the number of grid nodes is doubled in each of the two directions. Similar results were obtained in computations with a simultaneous increase in the number of nodes with respect to both variables (by a factor of 2 and 4). For this reason, the minimum possible

TABLE 3

Values of ΔM in Different Cross Sections and at the Nozzle Axis for the Nozzle with Different Mach Numbers

M_{out}	ΔM, %		
	Cross section $x = 2.5$ m	Cross section $x = 3.5$ m	Axis 2.5–3.5 m
8	0.25	0.16	0.65
10	1.47	1.68	0.40
12	0.58	0.29	0.45
14	0.41	0.29	0.02

TABLE 4

Values of Δ M in Different Cross Sections and at the Nozzle Axis for the Nozzle with Different Mach Numbers

M_{out}	ΔM, %		
	Cross section $x = 3.4$ m	Cross section $x = 3.9$ m	Axis 3.4–4.4 m
14	0.69	0.39	0.43
16	0.60	0.17	0.67
18	0.77	0.70	0.84
20	0.32	0.29	0.49

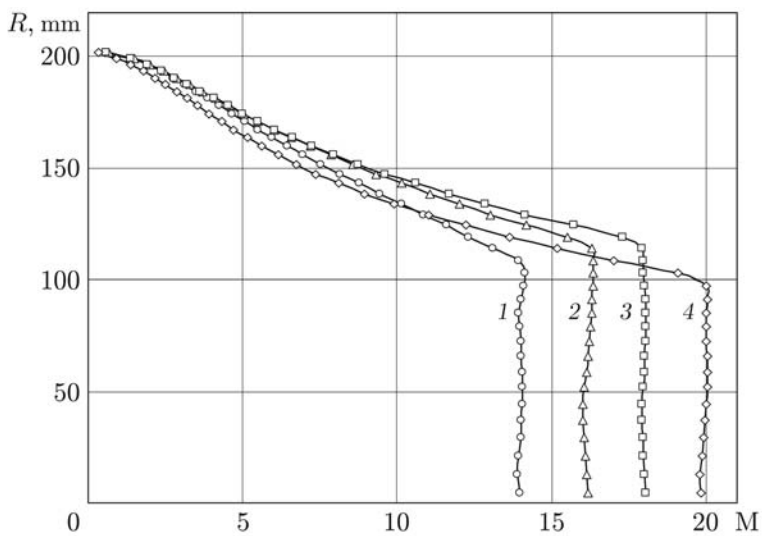

Fig. 6. Computed distributions of the Mach number at the exit of a multimodal nozzle with different output Mach numbers ($x = 3.9$ m): $M_{out} = 14$ (1), 16 (2), 18 (3), and 20 (4).

number of nodes was chosen for further computations, which is important for decreasing the computation time in solving optimization problems. For the same reason, the shape of the sonic line at the nozzle entrance was assumed to be straight, which allowed us to avoid computing the subsonic and transonic parts of the nozzle. Thus, the optimization problems posed could be solved with acceptable accuracy, which is confirmed by the good agreement between the experimental and computed results.

Design of the Multimodal Nozzle No. 1. Based on the technology developed, we designed a new multimodal nozzle for the Mach numbers $M_{out} = 8$, 10, 12, and 14. The nozzle constructed by the method of characteristics for a uniform distribution of the Mach number $M_{out} = 15.7$ was considered as the initial approximation in designing the basic contour for the Mach number $M_{out} = 14$. The resultant contour defined by a large number of points (up to 600) was approximated by a cubic spline on a small (up to 10) number of nonuniformly distributed reference points with the mean absolute and relative error of the order of 10^{-4}. This allowed us to vary the contour with a moderate number of parameters in solving optimization problems and, as a consequence, to reduce the time expenses in computations of the viscous flow in the nozzle. Then, the initial contour was modified by means of solving the optimization problem. During optimization and design, the viscous flow was again computed with the use of the Fluent CFD package integrated with a package for optimization problems. A functional equal to the sum of the root-mean-square deviations of the Mach number from its average value in a given flow area was minimized. Gradientless methods of the search with adaptation were used for functional minimization [11]. Constraints (e.g., the condition of monotonicity of the curve defining the contour and existence of no more than one inflection) set in the form of equalities and inequalities were taken into account by the method of penalty functions. It was necessary to modify the initial contour designed in an inviscid formulation because the calculations of the flow with allowance for viscosity in hypersonic nozzles designed by the method of characteristics show that considerable oscillations of parameters along the nozzle axis could arise. An example of the modification that allowed axial oscillations of the Mach number to be appreciably reduced can be found in [12]. In what follows, it is the greater part of the modified nozzle that is unchanged in design of the multimodal nozzle.

Then, we calculated the coordinates of the contour of the replaceable (initial) nozzle parts providing the Mach numbers $M_{out} = 8$, 10, and 12. For this purpose, the optimization problem was solved in the viscous formulation with the following parameters: total length of the nozzle 3 m, output diameter 0.41 m, and output Mach number M_{out}. The throat diameter and the geometry of the initial part 0.32 m long were varied. The shape of the replaceable part smoothly adjacent to the fixed part of the nozzle corresponding to the optimal nozzle with the Mach number $M_{out} = 14$ was defined by a cubic spline on a nonuniform grid. The nozzles were designed for the average Reynolds numbers Re_{av} with allowance for possible parameters of the gas in the settling chamber (Table 2).

Figure 4 shows the distributions of the Mach number in the resultant nozzles for all the computed modes in the cross section $x = 3$ m. The characteristics of uniformity of the flow formed in this nozzle are summarized in Table 3. It is seen that the flow nonuniformity in the working area of the nozzle is 0.3–0.4% in the experiments with $M_{out} = 8$, 12, and 14 (except for $M_{out} = 10$). Figure 5 shows the shapes of the 0.32-m replaceable part of the nozzle for the Mach numbers $M_{out} = 8$, 10, and 12.

Design of the Multimodal Nozzle No. 2. The multimodal nozzle with the Mach numbers $M_{out} = 14$, 16, 18, and 20 was designed in a similar manner. The initial contour was constructed by the method of characteristics for the output Mach number $M_{out} = 24$. After modification of the initial contour, the optimization problem with the following specified parameters was solved: total nozzle length 3.9 m, output diameter 0.41 m, and output Mach number M_{out}. The throat diameter and the shape of the initial part 0.3 m long were varied.

Figure 6 shows the distributions of the Mach number for all calculated regimes in the nozzle exit section at $x = 3.9$ m. It is seen that the boundary layer occupies a fairly large part of the nozzle cross section. The characteristics of flow uniformity are given in Table 4, which shows that the flow nonuniformity in the working zone of the nozzle is 0.3–0.8% in wide ranges of operation parameters.

Conclusions. A computational technology is developed for solving the problem of design and optimization of supersonic and hypersonic nozzles, including the direct method of solving the problems on the basis of various models of the medium and numerical methods of integrating the viscous gas flow equations. Classes of functions that define the nozzle contour geometry and the dimension of the space of varied parameters are chosen. The formulated constraints, the functionals used, and the methods of solving the minimization problem allow obtaining meaningful solutions of multimodal nozzle design problems in a wide range of requirements. Two multimodal nozzles are designed for the AT-303 hypersonic wind tunnel, which are intended to operate in the Mach number ranges $M_{out} = 8$–14 and $M_{out} = 14$–20 and to satisfy imposed requirements.

The authors are grateful to A. N. Kraiko who proposed the scheme of the solution described on page 220.

REFERENCES

1. O. N. Katskova, *Calculation of Equilibrium Gas Flows in Supersonic Nozzles* [in Russian], Comput. Center, Acad. of Sci. of the USSR, Moscow (1964).
2. A. P. Byrkin and V. P. Verkhovskii, “Supersonic axisymmetric contoured nozzle for a wind tunnel,” Russian Patent No. 1528116, MPK G 01 M9/04, Publ. 09.15.94.
3. V. M. Galkin and V. I. Zvegintsev, “Construction of a multimodal axisymmetric nozzle for a hypersonic wind tunnel,” in: *Fundamental and Applied Problems of Modern Mechanics,* Proc. IV Conf., Izd. Tomsk. Univ., Tomsk (2004), pp. 293–294.
4. V. M. Galkin, “Some schemes of a multimodal supersonic wind tunnel,” in: *Fundamental and Applied Problems of Modern Mechanics,* Proc. VI Conf., Izd. Tomsk. Univ., Tomsk (2008), pp. 350–351.
5. Yu. S. Volkov and V. M. Galkin, “Choice of approximations in direct problems of nozzle design,” *Zh. Vychisl. Mat. Mat. Fiz.*, **47**, No. 5, 923–936 (2007).

6. J. Dennis (Jr.) and R. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice-Hall, Englewood Cliffs (1983, 1986).
7. R. W. MacCormack, "The effect of viscosity in hyperbolicity impact cratering," AIAA Paper No. 354 (1969).
8. J. Korte, "Aerodynamic design of axisymmetric hypersonic wind-tunnel nozzle using a least-squares/parabolized Navier–Stokes procedure," *J. Spacecraft Rockets*, **29**, No. 5, 685–691 (1992).
9. S. M. Aulchenko and V. P. Zamuraev, "Design of a multi-regime nozzle by means of a direct method of numerical optimization," in: *Proc. 12th Int. Conf. on Methods of Aerophysical Research* (Novosibirsk, June 28–July 3, 2004), Inst. Theor. Appl. Mech., Sib. Div., Russ. Acad. of Sci., Novosibirsk (2004), pp. 15–18.
10. A. M. Kharitonov, V. I. Zvegintsev, L. G. Vasenev, et al., "Characteristics of the AT-303 hypersonic wind tunnel. Part 1. Velocity fields," *Thermophys. Aeromech.*, **13**, No. 1, 1–16 (2006).
11. A. F. Latypov and Yu. V. Nikulichev, "Specialized complex of optimization problems," Preprint No. 15-85, Inst. Theor. Appl. Mech., Sib. Div., Acad. of Sci. of the USSR, Novosibirsk (1985).
12. S. M. Aulchenko, V. M. Galkin, V. I. Zveginzev, and A. N. Shiplyuk, "Design of a multimode axisymmetric nozzle for a hypersonic wind tunnel by methods of numerical optimization," in: *Proc. 14th Int. Conf. on Methods of Aerophysical Research* (Novosibirsk, June 30–July 6, 2008), Inst. Theor. Appl. Mech., Sib. Div., Russ. Acad. of Sci., Novosibirsk (2008); CD-ROM, ISBN 978-5-98901-040-0.